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On Interval Valued Intuitionistic (α, β)-Fuzzy H_v-Subgroups

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Abstract The notion of intuitionistic fuzzy sets is a generalization of the notion of fuzzy sets which is introduced by Atanassov. In this paper we give the concept of an interval valued intuitionistic (α, β)-fuzzy H_v-subgroups of an H_v-groups by using the notion of "belongingness$(\in)$" and "quasi-coincidence(q)" of fuzzy points with fuzzy sets, where $\alpha \in \{\in, q\}$, $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and, then we give related properties of these notions.

Mathematics Subject Classification: 20N20

Keywords: H_v-group, H_v-subgroup, fuzzy H_v-group, fuzzy H_v-subgroup, interval valued intuitionistic (α, β)-fuzzy H_v-subgroup.

1. INTRODUCTION

Marty [1] introduced the concept of hyperstructures in 1934. Hyperstructures have many applications to several branches of pure and applied sciences. Vougiouklis [2] introduced the notion of H_v-structures, and Davvaz [3] surveyed the theory of H_v-structures. After the introduction of fuzzy sets by Zadeh [4], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [5] is one among them. For more details on intuitionistic fuzzy sets, we refer the reader to [6, 7].

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [14], played a vital role to generate some different types of fuzzy subgroups. Bhakat and Das [8, 9] gave the concepts of (α, β)-fuzzy subgroups by using the notion of "belongingness $(\in)$" and "quasi-coincidence (q)" between a fuzzy point and a fuzzy subgroup, where α, β are any two of $\{\in, q, \in \vee q, \in \wedge q\}$ with $\alpha \neq \in \wedge q$, and introduced the concept of an $(\in, \in \vee q)$-fuzzy subgroup. Yuan, Li et al. [10] redefined (α, β)-intuitionistic fuzzy subgroups. M. Asghari-Larimi [15] gave intuitionistic (α, β)-fuzzy H_v-submodules. Then Sinha and Dewangan [16] has given the concept of intuitionistic (α, β)-fuzzy H_v-subgroups. Now this paper continues this line of research for interval valued intuitionistic (α, β)-fuzzy H_v-subgroups of H_v-groups.

The whole paper is arranged in following style. Some fundamental definitions on H_v-structures and fuzzy sets are explored in section 2. We define interval valued intuitionistic (α, β)-fuzzy H_v-subgroups and establish some useful theorems in section 3.

2. BASIC DEFINITIONS

In order to prove further results we are giving some basic definitions.

Definition 2.1 [11] Let X be a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X.

Definition 2.2 [11] An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \lambda_A(x) \le 1$ for all $x \in X$. We shall use the symbol $A = \{\mu_A, \lambda_A\}$ for the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$.

Definition 2.3 [12] Let G be a non–empty set and $* : G \times G \to \wp^*(G)$ be a hyperoperation, where

$\wp^*(G)$ is the set of all the non-empty subsets of G. Where $A*B=\bigcup_{a\in A,\, b\in B} a*b,\ \ \forall A, B\subseteq G.$

The $*$ is called weak commutative if $x*y\cap y*x\neq\phi,\quad \forall x, y\in G.$
The $*$ is called weak associative if $(x*y)*z\cap x*(y*z)\neq\phi,\quad \forall x, y, z\in G.$

A hyperstructure $(G,*)$ is called an H_v–group if
(i) $*$ is weak associative.
(ii) $a*G=G*a=G,\quad \forall a\in G$ (Reproduction axiom).

Definition 2.4 [13] Let G be a hypergroup (or H_v-group) and let μ be a fuzzy subset of G. Then μ is said to be a fuzzy subhypergroup (or fuzzy H_v-subgroup) of G if the following axioms hold:

$$(i)\min\{\mu(x),\mu(y)\}\leq \inf_{\alpha\in x*y}\{\mu(\alpha)\},\forall x, y\in G$$

(ii) For all $x, a\in G$ there exists $y\in G$ such that $x\in a*y$ and $\min\{\mu(a),\mu(x)\}\leq\{\mu(y)\}.$

Definition 2.5 [5] Let $A=\{\mu_A,\lambda_A\}$ and $B=\{\mu_B,\lambda_B\}$ be intuitionistic fuzzy sets in X. Then

$(1)\ A\subseteq B\Leftrightarrow \mu_A(x)\leq\mu_B(x)\ and\ \lambda_A(x)\leq\lambda_B(x)$ $\forall x\in X$,

$(2)\ A^c=\{(x,\lambda_A(x),\mu_A(x)):x\in X\}$,

$$(3)\ A\cap B=\left\{\begin{matrix}(x,\min\{\mu_A(x),\mu_B(x)\},\\ \max\{\lambda_A(x),\lambda_B(x)\}):x\in X\end{matrix}\right\},$$

$$(4)\ A\cup B=\left\{\begin{matrix}(x,\max\{\mu_A(x),\mu_B(x)\},\\ \min\{\lambda_A(x),\lambda_B(x)\}):x\in X\end{matrix}\right\}.$$

Definition 2.6 [8] Let μ be a fuzzy subset of R. If there exist a $t\in(0,1]$ and an $x\in R$ such that

$$\mu(y)=\begin{cases}t & if\ y=x\\ 0 & if\ y\neq x\end{cases}$$

Then μ is called a fuzzy point with support x and value t and is denoted by x_t.

Definition 2.7 [8] Let μ be a fuzzy subset of R and x_t be a fuzzy point.

If $\mu(x)\geq t$, then we say x_t belongs to μ, and write $x_t\in\mu$.
If $\mu(x)+t>1$, then we say x_t is quasi-coincident with μ, and write $x_t q\mu$.
$x_t\in\vee q\mu\Leftrightarrow x_t\in\mu$ or $x_t q\mu$.
$x_t\in\wedge q\mu\Leftrightarrow x_t\in\mu$ and $x_t q\mu$.

In this paper unless otherwise stated, α and β will denote any one of $\in, q, \in\vee q$ or $\in\wedge q$ with $\alpha\neq\in\wedge q$, which was introduced by Bhakat and Das [9].

By taking the notations as taken by [17], an interval number $\tilde{a}$ we mean an interval $\left[a^-,a^+\right]$ where $0\leq a^-\leq a^+\leq 1.$ The set of all interval numbers is denoted by $D[0,1]$. We also identify the interval $[a,a]$ by the number $a\in[0,1]$.

For the interval numbers $\tilde{a}_i=\left[a_i^-,a_i^+\right]\in D[0,1], i\in I,$ we define

$$\max\left\{\tilde{a}_i,\tilde{b}_i\right\}=\left[\max\left(a_i^-,b_i^-\right),\max\left(a_i^+,b_i^+\right)\right],$$

$$\min\left\{\tilde{a}_i,\tilde{b}_i\right\}=\left[\min\left(a_i^-,b_i^-\right),\min\left(a_i^+,b_i^+\right)\right],$$

$$\inf\tilde{a}_i=\left[\bigwedge_{i\in I}a_i^-,\bigwedge_{i\in I}a_i^+\right],\sup\tilde{a}_i=\left[\bigvee_{i\in I}a_i^-,\bigvee_{i\in I}a_i^+\right]$$

and put

$(1)\ \tilde{a}_1\leq\tilde{a}_2\Leftrightarrow a_1^-\leq a_2^-$ and $a_1^+\leq a_2^+$,

$(2)\ \tilde{a}_1=\tilde{a}_2\Leftrightarrow a_1^-=a_2^-$ and $a_1^+=a_2^+$,

$(3)\ \tilde{a}_1<\tilde{a}_2\Leftrightarrow \tilde{a}_1\leq\tilde{a}_2$ and $\tilde{a}_1\neq\tilde{a}_2$,

$(4)\ k\tilde{a}=\left[ka^-,ka^+\right]$, whenever $0\leq k\leq 1$.

It is clear that $\left(D[0,1],\leq,\vee,\wedge\right)$ is a complete lattice with $0=[0,0]$ as least element and $1=[1,1]$ as greatest element.

By an interval valued fuzzy set F on X we mean the set $F=\left\{\left(x,\left[\mu_F^-(x),\mu_F^+(x)\right]\right):x\in X\right\}$.

Where μ_F^- and μ_F^+ are fuzzy subsets of X such that $\mu_F^-(x)\leq\mu_F^+(x)$ for all $x\in X$.

Put $\tilde{\mu}_F(x)=\left[\mu_F^-(x),\mu_F^+(x)\right]$.

Then $F=\{(x,\tilde{\mu}_F(x)):x\in X\}$,
Where $\tilde{\mu}_F : X \to D[0,1]$.

If A, B are two interval valued fuzzy subsets of X, then we define $A\subseteq B$ if and only if for all $x\in X$, $\mu_A^-(x)\le\mu_B^-(x)$ and $\mu_A^+(x)\le\mu_B^+(x)$, $A=B$ if and only if for all $x\in X$, $\mu_A^-(x)=\mu_B^-(x)$ and $\mu_A^+(x)=\mu_B^+(x)$.
Also, the union, intersection and complement are defined as follows: let A; B be two interval valued fuzzy subsets of X, then

$$A\cup B=\left\{\left(x,\begin{bmatrix}\max\{\mu_A^-(x),\mu_B^-(x)\},\\ \max\{\mu_A^+(x),\mu_B^+(x)\}\end{bmatrix}\right):x\in X\right\},$$

$$A\cap B=\left\{\left(x,\begin{bmatrix}\min\{\mu_A^-(x),\mu_B^-(x)\},\\ \min\{\mu_A^+(x),\mu_B^+(x)\}\end{bmatrix}\right):x\in X\right\},$$

$$A^c=\{(x,[\{1-\mu_A^-(x),1-\mu_A^+(x)\}]):x\in X\}.$$

According to Atanassov an interval valued intuitionistic fuzzy set on X is defined as an object of the form $A=\{(x,\tilde{\mu}_A(x),\tilde{\lambda}_A(x)):x\in X\}$, where $\tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x)$ are interval valued fuzzy sets on X such that $0\le\sup\tilde{\mu}_A(x)+\sup\tilde{\lambda}_A(x)\le 1$ for all $x\in X$.

For the sake of simplicity, in the following such interval valued intuitionistic fuzzy sets will be denoted by $A=(\tilde{\mu}_A,\tilde{\lambda}_A)$.

3. INTERVAL VALUED INTUITIONISTIC (α,β) - fuzzy H_v-subgroups

In this section we give the definition of interval valued intuitionistic (α,β)-fuzzy H_v-subgroup and prove some related results.

Definition 3.1 Let G be a H_v-group. An intuitionistic fuzzy set $A=\{\mu_A,\lambda_A\}$ of G is called intuitionistic fuzzy H_v-subgroup of G if the following axioms hold:

(i) $\min\{\mu(x),\mu(y)\}\le\inf_{\alpha\in x*y}\{\mu(\alpha)\},\forall x,y\in G.$

(ii) For all $x,a\in G$ there exists $y\in G$ such that $x\in a*y$ and $\min\{\mu(a),\mu(x)\}\le\{\mu(y)\}$.

(iii) $\sup_{\alpha\in x*y}\{\lambda_A(\alpha)\}\le\max\{\lambda_A(x),\lambda_A(y)\},\forall x,y\in G.$

(iv) For all $x,a\in G$ there exists $y\in G$ such that $x\in a*y$ and $\{\lambda_A(y)\}\le\max\{\lambda_A(a),\lambda_A(x)\}$.

Definition 3.2 An interval valued intuitionistic fuzzy set $A=\{\tilde{\mu}_A,\tilde{\lambda}_A\}$ in G is called an interval valued intuitionistic (α,β)-fuzzy H_v-subgroup of G if for all $t,r\in(0,1]$,

(1) $\forall x,y\in G,\quad x_t,y_r\alpha\,\tilde{\mu}_A\Rightarrow z_{t\wedge r}\beta\,\tilde{\mu}_A$ for all $z\in x\cdot y$,

(2) $\forall x,a\in G,\quad x_t,a_r\alpha\,\tilde{\mu}_A\Rightarrow y_{t\wedge r}\beta\,\tilde{\mu}_A$ for some $y\in G$ with $x\in a\cdot y$,

(3) $\forall x,y\in G,\quad x_t,y_r\overline{\alpha}\,\tilde{\lambda}_A\Rightarrow z_{t\wedge r}\overline{\beta}\,\tilde{\lambda}_A$ for all $z\in x\cdot y$,

(4) $\forall x,a\in G,\quad x_t,a_r\overline{\alpha}\,\tilde{\lambda}_A\Rightarrow y_{t\wedge r}\overline{\beta}\,\tilde{\lambda}_A$ for some $y\in G$ with $x\in a\cdot y$.

Lemma 3.3 Let $A=\{\tilde{\mu}_A,\tilde{\lambda}_A\}$ be an interval valued intuitionistic fuzzy set in G. Then for all $x\in G$ and $r\in(0,1]$, we have

(1) $x_t q\tilde{\mu}_A\Leftrightarrow x_t\,\overline{\in}\,\tilde{\mu}_A^c.$

(2) $x_t\in\vee q\tilde{\mu}_A\Leftrightarrow x_t\overline{\in\wedge q}\,\tilde{\mu}_A^c.$

Proof (1) Let $x\in G$ and $r\in(0,1]$. Then, we have

$$\begin{aligned}x_t q\tilde{\mu}_A &\Leftrightarrow \tilde{\mu}_A(x)+t>1\\ &\Leftrightarrow 1-\tilde{\mu}_A(x)<t\\ &\Leftrightarrow \tilde{\mu}_A^c(x)<t\\ &\Leftrightarrow x_t\,\overline{\in}\,\tilde{\mu}_A^c.\end{aligned}$$

(2) Let $x\in G$ and $r\in(0,1]$. Then, we have

$$\begin{aligned}x_t\in\vee q\tilde{\mu}_A &\Leftrightarrow x_t\in\tilde{\mu}_A \text{ or } x_t q\tilde{\mu}_A\\ &\Leftrightarrow \tilde{\mu}_A(x)\ge t \text{ or } \tilde{\mu}_A(x)+t>1\\ &\Leftrightarrow 1-\tilde{\mu}_A^c(x)\ge t \text{ or} 1-\tilde{\mu}_A^c(x)+t>1\\ &\Leftrightarrow x_t\overline{q}\tilde{\mu}_A^c \text{ or } x_t\,\overline{\in}\,\tilde{\mu}_A^c\end{aligned}$$

$$\Leftrightarrow x_t \overline{\in \wedge q} \tilde{\mu}_A^c.$$

Theorem 3.4 If $A=\{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an interval valued intuitionistic $(\in, \in)$-fuzzy H_v-subgroup of $G,$ then $A=\{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an interval valued intuitionistic fuzzy H_v-subgroup of $G.$

Proof (1) Suppose $x, y \in G$ and $\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(y) = t.$ Then $x_t, y_t \in \tilde{\mu}_A.$ By condition (1) of definition 3.2, we have $z_t \in \tilde{\mu}_A, \quad \forall z \in x \cdot y,$ and so $\tilde{\mu}_A(z) \geq t, \quad \forall z \in x \cdot y.$

Consequently $\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(y) = t \leq \bigwedge_{z \in x \cdot y} \tilde{\mu}_A(z)$ for all $x, y \in G.$

(2) Now assume $x, a \in G$ and $\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(a) = t.$ Then $x_t, a_t \in \tilde{\mu}_A.$ It follows from condition (2) of definition 3.2 that $y_t \in \tilde{\mu}_A,$ for some $y \in G$ with $x \in a \cdot y.$

Thus $y_t \in \tilde{\mu}_A,$ for some $y \in G$ with $x \in a \cdot y.$

So, for all $x, a \in G,$ there exist $y \in G$ such that $x \in a \cdot y$ and $\tilde{\mu}_A(x) \wedge \tilde{\mu}_A(a) = t \leq \tilde{\mu}_A(y).$

(3) Let $x, y \in G$ and $\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(y) = s.$ If $s=1$, then $\tilde{\lambda}_A(z) \leq 1 = s$ for all $z \in x \cdot y.$ It is easy to see that $\bigvee_{z \in x \cdot y} \tilde{\lambda}_A(z) \leq \tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(y)$ for all $x, y \in G.$

If $s<1$ there exists a $t \in (0,1]$ such that $\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(y) = s < t$

Then $x_t, y_t \overline{\in} \tilde{\lambda}_A.$ By condition (3) of definition 3.2, we have $z_t \overline{\in} \tilde{\lambda}_A, \quad \forall z \in x \cdot y$ and so $\tilde{\lambda}_A(z) < t.$

Consequently $\bigvee_{z \in x \cdot y} \tilde{\lambda}_A(z) \leq \tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(y)$ for all $x, y \in G.$

(4) Now let $x, a \in G$ and $\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(a) = s.$ If $s<1$, there exists a $t \in (0,1]$ such that $\tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(a) = s < t.$

Then $x_t, a_t \overline{\in} \tilde{\lambda}_A.$ By condition (4) of definition 3.2, we have $y_t \overline{\in} \tilde{\lambda}_A$ for some $y \in G$ with $x \in a \cdot y$

Hence $\tilde{\lambda}_A(y) < t$ and $\tilde{\lambda}_A(z) < t.$

Thus $\tilde{\lambda}_A(y) \vee \tilde{\lambda}_A(z) < t.$ This implies that for all $x, a \in G,$ there exist $y \in G$ such that $x \in a \cdot y$ and $\tilde{\lambda}_A(y) \leq \tilde{\lambda}_A(x) \vee \tilde{\lambda}_A(a).$ If $s=1$ the proof is obvious.

Theorem 3.5 If $A=\{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an interval valued intuitionistic $(\in, \in \vee q)$ and $(\in, \in \wedge q)$-fuzzy H_v-subgroup of $G,$ then $A=\{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an interval valued intuitionistic fuzzy H_v-subgroup of $G.$

Proof The proof is similar to the proof of Theorem 3.4.

Theorem 3.6 If $\square A=\left\{\tilde{\mu}_A, \tilde{\mu}_A^c\right\}$ is an interval valued intuitionistic (α, β)-fuzzy H_v-subgroup of G if and only if $\square A=\left\{\tilde{\mu}_A, \tilde{\mu}_A^c\right\}$ is an interval valued intuitionistic (α', β')-fuzzy H_v-subgroup of $G,$ where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in \vee q, \in \wedge q\}.$

Proof We only prove the case of $(\alpha, \beta) = (\in, \in \vee q).$ The others are analogous. Let $\square A=\left\{\tilde{\mu}_A, \tilde{\mu}_A^c\right\}$ be an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-subgroup of $G.$

(1) Let $x, y \in G$ and $t, r \in (0,1]$ be such that $x_t, y_r q \tilde{\mu}_A.$ It follows from Lemma 3.3 that $x_t, y_r \overline{\in} \tilde{\mu}_A^c.$ Since $\tilde{\mu}_A^c$ is an anti $(\in, \in \vee q)$-fuzzy H_v-subgroup of $G.$ Thus by condition (3) of definition 3.2, we have

$$z_{t \wedge r} \overline{\in \vee q} \tilde{\mu}_A^c \text{ for all } z \in x \cdot y.$$

By Lemma 3.3, this is equivalence with

$$z_{t \wedge r} \in \wedge q \tilde{\mu}_A \text{ for all } z \in x \cdot y.$$

Thus condition (1) of definition 3.2 is valid.

(2) Suppose that $x, a \in G$ and $t, r \in (0,1]$ be such that $x_t, a_r q \tilde{\mu}_A.$ By Lemma 3.3, we have $x_t, a_r q \tilde{\mu}_A$ iff $x_t, a_r \overline{\in} \tilde{\mu}_A^c.$ By hypotheses, $\tilde{\mu}_A^c$ is an anti $(\in, \in \vee q)$-fuzzy H_v-subgroup of $G.$ Thus by condition (4) of definition 3.2, we have

$y_{t\wedge r} \overline{\in \vee q} \tilde{\mu}_A^c$ for some $y \in G$ with $x \in a \cdot y$.
It follows from Lemma 3.2 that
$y_{t\wedge r} \in \wedge q \tilde{\mu}_A$ for some $y \in G$ with $x \in a \cdot y$.
Thus condition (2) of definition 3.2 is valid.

(3) Let $x, y \in G$ and $t, r \in (0,1]$ be such that $x_t, y_r \overline{q} \tilde{\mu}_A^c$. It follows from Lemma 3.3 that $x_t, y_r \overline{q} \tilde{\mu}_A^c$ iff $x_t, y_r \in \tilde{\mu}_A$. Since $\Box A = \{\tilde{\mu}_A, \tilde{\mu}_A^c\}$ is an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-subgroup of G. Thus by condition (1) of definition 3.2, we have
$z_{t\wedge r} \in \vee q \tilde{\mu}_A$ for all $z \in x \cdot y$.
By Lemma 3.2, this is equivalence with
$z_{t\wedge r} \overline{\in \wedge q} \tilde{\mu}_A^c$ for all $z \in x \cdot y$.
Thus condition (3) of definition 3.2 is valid.

(4) Suppose that $x, a \in G$ and $t, r \in (0,1]$ be such that $x_t, a_r \overline{q} \tilde{\mu}_A^c$. This is equivalence with $x_t, a_r \in \tilde{\mu}_A$. By hypotheses, $\tilde{\mu}_A$ is an $(\in, \in \vee q)$-fuzzy H_v-subgroup of G. Thus by condition (2) of definition 3.2, we have
$y_{t\wedge r} \in \vee q \tilde{\mu}_A$ for some $y \in G$ with $x \in a \cdot y$.
It follows from Lemma 3.3 that
$y_{t\wedge r} \overline{\in \wedge q} \tilde{\mu}_A^c$ for some $y \in G$ with $x \in a \cdot y$.
Thus condition (4) of definition 3.2 is valid.

Theorem 3.7 If $\Diamond A = \{\tilde{\lambda}_A^c, \tilde{\lambda}_A\}$ is an interval valued intuitionistic (α, β)-fuzzy H_v-subgroup of G if and only if $\Diamond A = \{\tilde{\lambda}_A^c, \tilde{\lambda}_A\}$ is an interval valued intuitionistic (α', β')-fuzzy H_v-subgroup of G, where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$.
Proof The proof is similar to the proof of Theorem 3.6.

Theorem 3.8 If $A = \{\tilde{\mu}_A, \tilde{\lambda}_A\}$ is an interval valued intuitionistic (α, β)-fuzzy H_v-subgroup of G if and only if $\tilde{\mu}_A$ is an (α, β)-fuzzy H_v-subgroup of G and $\tilde{\lambda}_A^c$ is an (α', β')-fuzzy H_v-subgroup of G, where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$.
Proof We only prove the case of $(\alpha, \beta) = (\in, \in \vee q)$. The others are analogous. It is sufficient to show that, $\tilde{\lambda}_A^c$ is an $(q, \in \wedge q)$-fuzzy H_v-subgroup of G if and only if $\tilde{\lambda}_A$ is an anti $(\in, \in \vee q)$-fuzzy H_v-subgroup of G. This is true, because $x_t q \tilde{\lambda}_A \Leftrightarrow x_t \overline{\in} \tilde{\lambda}_A^c$ and
$x_t \in \wedge q \tilde{\lambda}_A \Leftrightarrow x_t \overline{\in \vee q} \tilde{\lambda}_A^c$, $\forall x \in G$ and $t \in (0,1]$.

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